THE
GEOMETRICAL SQUARE:
WITH THE USE THEREOF
IN
PLAIN and SPHERICAL
TRIGONOMETRIE.

Chiefly intended for the more easie finding of the HOUR and AZIMUTH.

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A DESCRIPTION OF THE SQUARE.

THE whole superficies is divided into four lesser Squares, by the Diameters F G and H I.

Each of the 4 Semidiameters E F, E H, E G, E I, are divided as the lines of Sines upon the Sector, the Semidiameters being the whole Sine, And through the parts of each Semidiameter are drawn right lines perpendicular thereunto, quite over the face of the whole Square every 10th, 5th, &c. are to be distinguished from the rest, for the more easie and speedy account.

Upon the limb are inserted several Scales, for several uses. The edges of these Scales bordering close upon the sides of the inner Square, that it may be discerned which lines and parts of the Scales doe butt one upon the other.

On the sides A B, A C, are inscribed Scales of equal parts, the whole being divided into 10, and subdivided as quantity will give leave. The parts are numbred by 1. 2. 3. 4. 5. 6. 7. 8. 9. 10, and may stand, either for 1. 2. 3, &c. or for 10. 20. 30, &c. or for 100. 200. 300, &c. as occasion requireth.

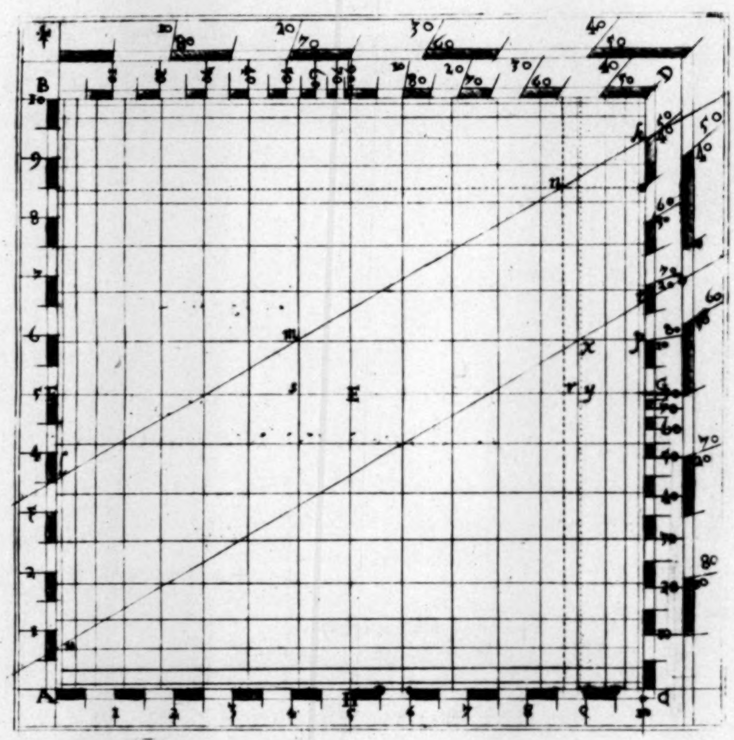
To the lines B I and C G, are annexed Scales of right sines whose beginning is at B and C, and the end at I and G, numbred by 10. 20, &c. to 90.

The

The other parts I D, G D, have Scales of Tangents from o gr. to 45 gr. numbred either from G to D, and ſo to I. Or from I to D, and ſo to G, or rather both wayes, by 10, 20, 30, &c. till 90, and theſe diviſions have reſpect to the Center E.

Laſtly, a Scale of larger Tangents, lying behind theſe laſt named, Parallel to the ſides BD, C D, beginning o gr. from B and C, and ſo proceeding to 45 gr. in D and ending in 90 gr. at C and B, accounted both wayes. Theſe have reſpect to their Center A.

Τὸ μέγα βιβλίον ἴσον τῷ μεγάλῳ κακῷ.



There is further added a threed and plummet, which is to be uſed in every practice, and muſt be in length equal to the lines A F, and F G. And if the threed be found inconvenient in practice, becauſe it will take up the uſe of both hands, there may inſtead of it, be uſed a little Bowe, the threed of it being at the leaſt equal to A D, which will perform the office of the other threed by the help of one hand only, or a ſtraight ruler may ſerve, if it be thought convenient for that purpoſe.

If the Square be applyed to the obſervation of Angles, it may

may be fitted thereto one of these two wayes, *Either by placing* two sights upon the side of the Square, one upon the Center A, the other upon the line A B, which issueth out of the Center A. *And a running sight* contrived upon the utter edge of the Instrument to move from B to C by D forward, and so from C to B by D, backward again; *Or else* if this be thought inconvenient, or not feasible, because of the sights turning over at the Angle D, then this moveable sight may goe onely upon one of the sides B D or C D. And for that purpose the sight at A, is to stand precisely upon the Center, and both the sides A B, A G must have sights there fixed, as precisely, upon their lines that come from A.

Of the use of the Square in General for the Solution of Spherical Triangles.

In any Spherical Triangle whatsoever.

¶ *By having the Legs and Base, to find the Vertical Angle.*

THe Angle given or sought is the Vertical Angle, The sides comprehending it are the legs. The side subtending it is the Base.

From the top of the Square, count the sum of the legs upon one side, the difference of them on the other side, To this sum and difference apply the threed, Then from the same top of the Square count the base also, And mark where it cuts the threed, for the line passing through the intersection, and standing Square to the top, (if it be numbred from that side of the Square whereon the difference of the legs was counted) gives the Vertical Angle required.

This is the general manner of work for this Proposition, which may be illustrated by these particulars.

FIRST,

Having the Latitude of the place, the Declination and Altitude of the Sun, to find the Hour of the day.

BY the declination of the Sun, may be had his distance from the elevated Pole, By subtracting it from 90 gr. when the Declination is of the same denomination with the said Pole,

Pole; Or by adding the Declination to 90 gr. when the Declination and elevated Pole are of several denominations.

In this case, we have the three sides of a Spherical Triangle given, and an Angle sought.

The two legs are *The complement of the Latitude*, and *The Suns distance from the Pole*. The base is, *The complement of the Suns Altitude :* The Angle is the *Hour* required, which must be accounted from the Coast of a contrary name, to the elevated Pole.

According then to the former general prescript, and this particular declaration, *For the hour take the sum and difference of the complement of the Latitude, and of the Suns distance from the Pole, and from the top of the Square, upon one side, count the difference, the sum on the other, to these terms apply the threed; Then from the top of the Square also, count the complement of the Suns Altitude, and where it cuts the threed, the line that crosseth it Square in the same point (being reckoned from that side whereon the difference of the legs was counted) gives the hour from the Meridian or noon.*

To make it plain by an Example.

In a North Latitude of 52 gr. 30 min. the Sun declining 20 gr. to the North, the Altitude of the Sun being by observation 43 gr. I would know the Hour of the day. The legs of this Triangle are the complements of the latitude and declination, that is 37 gr. 30 min. and 70 gr. 0 min. The sum of them is 107 gr. 30 min. their difference is 32 gr. 30 min. Then from the top of the Square at D upon the side DG, I reckon this difference 32 gr. 30 min. downward to *k*. And on the other side of the Square from the top at B, I also count the sum of the legs 107 gr. 30 min. downward to *l*. To *k* and *l*. I apply the threed. Which done from the top of the Square, again, I count the base 47 gr. the complement of 43 gr. the altitude observed, downward also to o, and the line that there meets me, I follow till it cut the threed, which is at *n*, and the line that there ariseth Square to it is *n r*. *I say now that n r*, if it be counted from the side DC whereon the difference of the legs was counted, shall give 44gr. 8 min. which turned into hours and minutes of an hour, (allowing 15 gr. to an hour; and 15 min. of a degree to one minute of an hour) will make two hours and 56 $\frac{1}{2}$ min. from the

the Meridian or South, And such is the Hour for that *Latitude, Altitude*, and *Declination*.

So also, If in the same Latitude and distance of the Sun from the Pole, but in the altitude of 10 gr. I would know *The houre of the day.* Here because the legs, that is, the complement of the latitude and the distance from the Pole, are the same, therefore the same position of the threed remaines still, I therefore onely reckon the base (as before) which here is 80 gr. from D to *p*, then I follow the line *p*, till it cuts the threed at *m*, and the line there arising is *m s*, which counted from D C, whereon the difference of the legs was reckoned, shall give 99 gr. 50 min. that is 6 hours 39 $\frac{1}{3}$ min. of an hour from the Meridian or South.

Another Example.

In the same latitude of 52 gr. 30 min. let the declination of the Sun be 20 gr. to the South, where his distance from the elevated Pole is 110 gr. and let the altitude of the Sun be by observation 10 gr. I require the *Hour*. The legs are 37 gr. 30 min. the complement of the latitude, And 110 gr. the Suns distance from the Pole. The sum of them is 147 gr. 30 min. The difference 72 gr. 30 min. which I count upon the sides of the Square down to *u* and *t*; and the base which is 80 gr. the complement of 10 gr. I count also from D to *p*, then I follow the line *p*, till it cut the threed at *x*, and the line there arising is *x y*, which counted from D C, whereon the difference of the legs was reckoned, shall give 38 grad. 56 min. that is, two hours and almost 36 min. of an houre from the Meridian or South.

Note, That the threed in this situation, shewes on the diameter of the Square (which in this case represents the Horizon) the *Semidiurnal* and *Seminocturnal* Arks, for where the threed crosseth the middle line, the line there arising, (counted from that side of the Square, whereon the difference was numbred) shewes the *Semidiurnal* ark, and counted from the other side, shewes the *Seminocturnal* ark.

Observe also, If you would known the *Crepusculum* or *Twilight*, the threed is to be placed as before, according to the sum and difference of the legs, and if you allow 18 gr. for the *Crepus-*

Crepusculin line (as they usually doe) the base will alway be 108 gr. which in the two first Examples will not touch the threed at all, and therefore in that latitude and parallel of the Sun, the twilight continues all night. But in the last Example you shall find the *Crepusculin* line to cut the threed, 6 hours and 15 min. from the Meridian, which shewes that the twilight begins at $5\frac{3}{4}$ a clock in the morning, and ends at $6\frac{1}{4}$ in the evening, and the rest of the time is dark night which is $11\frac{1}{2}$ hours.

If the sum of the the legs be more then 180 gr. that is, if it would reach beyond the bottom of the Square, you must when you have reckoned to the bottom, count upward back again till you have ended the whole sum.

SECONDLY,

Having the Latitude of the place, the Declination and Altitude of the Sun, To find the Azimuth of the Sun.

HEre also the 3 sides are given, the same with the former, and an Angle sought. The two legs are the Complements of the latitude, and Suns altitude, The base is the Suns distance from the Pole which is elevated above the Horizon. The angle sought is the Suns *Azimuth*, from that part of the Meridian, which is of the same denomination with the elevated Pole.

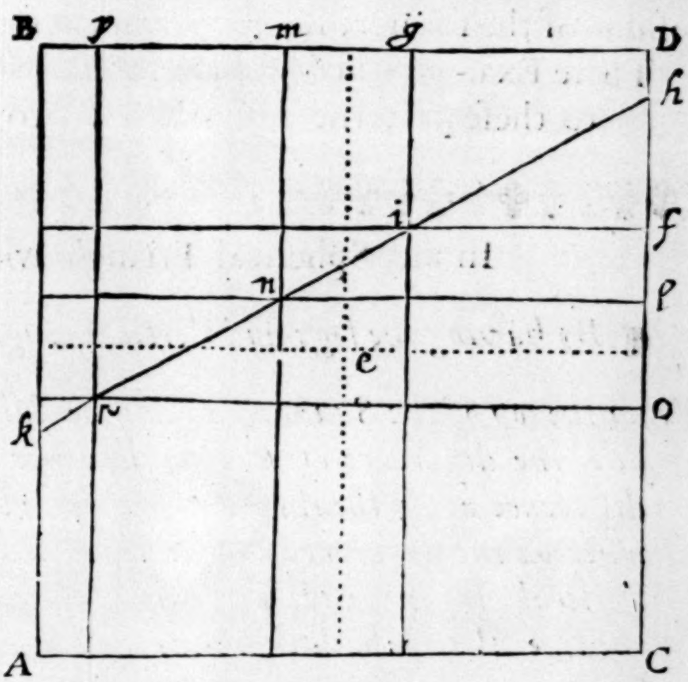
So then according to the former general prescript, and this particular declaration, for the *Azimuth*, doe thus.

Take the sum and difference of the Complements of the latitude and Suns altitude, and count from the top of the Square, the one upon one side, the other on the other side, and to these terms apply the threed; Then from the top of the Square also, count the Suns distance from the Pole, and where it doth crosse the threed, the line that there ariseth Square to the former, being reckoned from that side of the Square whereon the difference of the legs was counted, gives the Azimuth from that part of the Meridian which is of the same denomination with the elevated Pole, and counted from the other side, gives the Azimuth from the other coast.

To

To Illustrate it by an Example.

In a North latitude of 52 gr. 30 min. let the altitude of the Sun be 22 gr. and the declination 10 gr. Northerly, By these given I would know the Suns Azimuth, the two legs of the Triangle are the complements of the latitude and Suns altitude, that is 37 gr. 30 min. and 68 gr. the sum of them is 105 gr. 30 min. the difference is 30 gr. 30 min. The sum of them I count on the side B A, from the top at B down to *k*, The difference I count on the other side from D down to *h*, and to these points *k* and *h*, I apply the threed *k h*, And lastly, because the declination is 10 gr. North-ward in a North latitude, therfore his distance from the elevated Pole is 80 gr. which I count from the top D, down to *l*, and follow the line at *l*, till it meet with the threed at *n*, where I find the line *m n*, to crosse it also, which numbred from the side D C, whereon



the difference of the legs was numbred, gives 102 gr 38 m. the Azimuth from the North: And so also if it be accounted from the side B A, it gives the Azimuth from the South 77 gr. 22 min. the residue of the former, or the complement of it to 180 gr.

Another Example, In the same latitude and the same altitude, and therefore also the same situation, of the threed, let the declination be Northerly 23 $\frac{1}{2}$ gr. therefore the distance from the Pole will be 66 $\frac{1}{2}$ gr. which I count from D to *f*, and following the line *f* till it meet with the threed at *i*, I find the line *g i*, to crosse there also, which being counted from the side D C, whereon the difference of the legs was counted,

ſhewes 79 gr. 38 min. the Azimuth from the North, Or counted from the other ſide, gives the reſidue of the former, 100 gr. 22 min. The Azimuth from the South.

A third Example. In the ſame latitude and altitude, and therefore alſo in the ſame ſituation of the threed, let the declination of the Sun be 10 gr. to the South, then ſhall his diſtance from the elevated North Pole be 100 gr. and becauſe this 100 gr. is the baſe, I therefore count it from the top D, down to *o*, and following the line *o*, I find it to cut the threed at *r*, and the line *r p* there croſſing, ſhewes me from D C, (the ſide whereon the difference of the legs was counted) 146 gr. 32 min. for the Azimuth from the North, or if the ſame line be numbred from the ſide B A, it ſhewes 33 gr. 28 min. the reſidue of the former, for the Azimuth from the South.

These Examples may ſuffice for this kind, and according to theſe patternes, all others are to be framed.

In any Spherical Triangle whatſoever.

By having the legs and Vertical Angle, to find the Baſe.

From the top of the Square, count the ſum of the legs upon one ſide, the difference of them on the other ſide. To this ſum and difference apply the threed: Then from that ſide of the Square *whereon the difference of the legs was numbred, count the Vertical Angle given, and where it cuts the threed, mark the line that paſſeth there-through parallel to the top of the* Square, *for that line, counted from the top, gives the baſe required.*

This is general for all works of this kind, which may be illuſtrated in particular: thus,

Having the latitude of the place, the declination of the Sun, and the hour of the day, to find the altitude of the Sun, for that latitude, declination, and hour.

HEre have we the two legs of the Triangle, with the intercepted Vertical Angle, given, and the baſe ſought. The legs are the complement of the latitude, and the Suns diſtance, from the Pole; The angle intercepted is the hour

hour whose altitude we seek. And the base is the complement of the altitude sought for.

Wherefore by the former general prescipt, and this particular explication, we may attain to the thing required thus, as in the former practice, so here; *Apply the threed to the sum and difference of the complement of the latitude, and of the Suns distance from the Pole, Then reckon the hour given from the Meridian, from the side whereon the difference of the legs was counted, and where it crosseth the threed observe the line that passeth there-through parallel to the top of the Square, for that line reckoned from the top, shewes the base, that is, The complement of the altitude, or reckoned from the middle line of the Square, it gives the altitude it self of the Sun, for that parallel and Hour; And so the threed (which now represents the Suns parallel:) lying still, you may count the altitudes for all the rest of the hours for that parallel.*

For Example.

In a North latitude of 52 gr. 30 min. let the Sun decline 20 gr. to the North, so that his distance from the elevated North-pole will be 70 gr. which is one of the legs given, and the complement of the latitude, 37 gr. 30 min. is the other, The sum of them is 107 gr. 30 min. The difference is 32 gr. 30 min. This difference is the complement of the Suns Meridian altitude; and I count it from D the top of the Square, to *k*: (in the first figure) thereto applying one end of the threed And on the other side from B, I count the sum, 107 gr. 30 min. down to *l*, thereto applying the other end of the threed. The threed thus laid, resembles the Suns parallel, for that declination, Now from the side D *k*, whereon the difference was numbred, I count the Vertical Angle, As first 15 gr. for the first hour from the Meridian, either 11 in the morning, or one in the afternoon, and where it cuts the threed, I observe the other line there crossing also, which counted from the top, gives for the base, 34 gr. 32 min. the complement of the altitude required.

Or rather. Count it from the middle line, which in this case represents the Horizon, and then you shall have 55 gr. 28 m. the altitude it self, for that hour and parallel, So the second

hour from the Meridian (10 or 2) gives for the altitude 50 gr. 4 min. The third hour (9 or 3) gives 42 gr. 31 m. The fourth hour from the Meridian (8 or 4) gives 33 gr. 53 min. The fifth (7 or 5) gives 24 gr. 48 min. The sixth (6 in the morning and evening) gives 15 gr. 45 m. The seventh (5 in the morning, or 7 in the evening) gives 7 gr. 6 m. And so farre the Sun is above the Horizon in that parallel, and then begins to go down.

And observe further, That the threed thus placed taken in that part below the Horizon, gives the altitudes for the hours in the declination which is equal to this, but to a contrary coast; so that the threed in this situation, gives the altitudes for the declination of 20 grad. towards the South, for that part of the threed that is under the Horizon or middle line, is the Semi-nocturnal ark for the parallel lying 20 gr. from the Equinoctial Northward, and is therefore equal to the Semi-diurnal ark that belongs to the parallel which lies 20 gr. from the Equinoctial Southward, and is of like situation below the Horizon that the other is above, wherefore the depressions belonging to the hours in this, are the same with the altitudes of the same hours in the other. To go on then where we left, The next hour counted from the Meridian of the Winter parallel is the fourth, that is, either 8 in the morning, or 4 in the afternoon, and his depression is 0 gr. 50 min. The next hour the third from the Meridian (either 9 or 3) is depressed 7 gr. 39 m. The second hour, (10 or 2) is 12 gr. 57 m. The first hour (11 or 1,) is depressed in this North parallel 16 gr. 20 min. that is, it is elevated so much in the like South parallel. Thus of each two opposite parallels of declination may the altitudes be had at one and the same situation of the threed. But if the other way seeme plainer, do as before. Let the Sun in the same latitude decline 20 gr. to the South, his distance from the North elevated Pole, is then 110 gr. the sum of the complement of the latitude 37 gr. 30 min. and this distance is 147 gr. 30 min. The difference is 72 gr. 30 m. This difference I count from D to *t*, (in the first figure,) The sum I also account as before, from B to *u*, And to *t u*, I placed the threed, Now from the side D *t*, I count the hours as I did before, and find the altitude of 11 and 1, 16 gr. 20 min. of 10 and 2, 12 gr. 57 min. of 9 and 3, 7 gr. 39 min. of 8 and 4, 0 gr. 50 min. All the same that the former depressions were;

And if now you take the depressions of the hours upon the threed in this situation, you shall find them all the same that the altitudes in the former parallel of 20 gr. North declination were; So that ever, one side of the threed will afford the altitudes for the hours in any two opposite parallels.

The Meridian altitude is the complement of the difference of the legs, And in the opposite parallel it is the excesse of the sum of the legs above 90 gr. And as you have done for the Altitudes of the whole hours, so may you doe for their halves and quarters. Thus much for this also.

In any Spherical Triangle, whatsoever.

If the Proportions be in right Sines alone, they are resolved in this manner.

Count the first sine given (upon one of the sides of the lesser Square E I D G,) *from the Center* E, *and upon the line there arising count the second sine, whereto apply the threed, Then upon the same side with the first, count the third, and observe the line there arising, for from it doth the threed cut off the fourth sine required.*

This general may be illustrated in particular thus.

Having the greatest Declination of the Sun, and his distance from the next Equinoctial point; to find the Declination of the Sun for that distance.

THis particular belongs to the solution of a Rect-angled Spherical Triangle, yet the manner of the work in this is the same with the work belonging to the solution of the Obliquangled ones. The proportion stands thus; *As the radius, is to the sine of the greatest declination; So the sine of the Suns distance from the next Equinoctial point; to the sine of the declination of that point.*

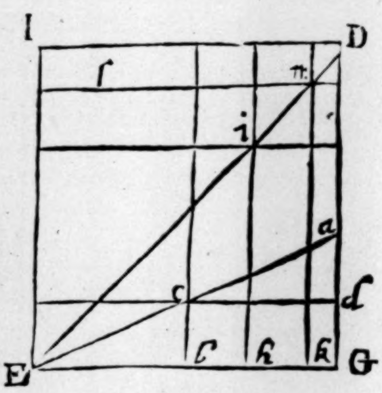
For an Example.

Let the distance from the Equinoctial be 30 gr. The greatest declination 23 gr. 30 min. I would know the declination for that distance of 30 gr. The Proportion is, As the radius, is

is to the ſine of 23 gr. 30 min. So the ſine of 30 degrees, to what ſine? Count E G for the radius, and upon G D the line there ariſing, reckon 23 deg. and 30 min. up to *a*, and thereto apply the threed, Then again upon E G, count *e b*, the ſine of 30 gr. and follow the line there ariſing which is *b c*, till it cut the threed at *c*, and the line *c d*, there croſſing alſo (being counted from E G) gives for the declination required, 11 gr. 30 min. So that the ſine of 11 gr. 30 min. is the fourth Proportional Sine to the former three.

By the Hour of the Day given, with the Suns diſtance from the elevated Pole, and the complement of his altitude above the Horizon, to find his Azimuth.

The Azimuth thus gotten is counted in North latitudes from the North, in South latitudes from the South. Let the hour be 3 from the meridian. The Suns diſtance from the Pole 66 gr. 30 min. The complement of the altitude, 44 gr. 20 m. By theſe things known, I would find the Azimuth. The proportion whereby it is wrought is this.



As the Sine of the hour, is to the Sine of the complement of the altitude; So is the ſine of the diſtance, to the ſine of the Azimuth. Wherefore upon the ſide of the Square E G, I count the ſine of 45 gr. to *b*, and upon the line there ariſing, I count the ſine of 44 gr. 20 min. up to *i*, and thereto apply the threed. Then upon the ſame ſide with the firſt I reckon the ſine of the Suns diſtance 66 gr. 30 min. from E to *k*, and following the line there ariſing till it meet with the threed at *n*, I find the line *n l*, to croſſe there alſo, which counted from E G, gives the ſine of 65 gr. for the fourth proportional, ſo that 65 gr. ſhewes the reſidue of the Angle required, that is to ſay, In our North latitude it ſhewes me the Azimuth from the South; becauſe the Angle of the Azimuth from the North is an obtuſe Angle, namely 115 gr. and the ſame ſine ſerves both to it and to 65 gr. his reſidue.

¶ *And here alſo is to be noted, That when any Proportion in right ſines alone is offered, and the radius is the firſt leader*

in

in the Proportion, that then I say, it may be resolved by the former kind of work, by the sum and difference, counting the complement of the arks of the two Sines *given, for the legs of the Triangle, and the ark of the radius or 90 gr. for the Vertical Angle, and the base found out to be the complement of the ark required.* As in the first Example,

The two middle arks were 30 gr. and 23 gr. 30 min. their complements are *60* gr. and *66* g. 30 min. The sum of these is 126 gr. 30 min. there difference 6 gr. 30 min. to this sum and difference, I apply the threed, as in the former Examples, and then count the Vertical Angle 90 gr. which fals in the middle line and where the threed cuts it, there is the quantity of the Declination, 11 gr. 30 min. as before: And these degrees are counted from the Center of the Square at E.

And thus may all others of this nature, having the radius in the first place, be absolved. And not onely these of sines alone, but with sines intermingled with Tangents also, If it so fall out that these Tangents be lesse then the radius, And if instead of their proper arks be taken the complements of the arks of sines equal unto those Tangents.

And thus much for Exemplifying in this kind also. Those that follow are appropriate to rectangled Spherical Triangles only.

In any Rectangled Spherical Triangle whatsoever.

¶ *If the Proportion stand between right sines (whereof the* Radius *is alway one) and Tangents, they are to be resolved in this manner.*

Upon one of the sides of the lesser square EIGD. *Count the first term, and upon the line there arising count the second, whereto apply the threed. Then upon the same side whereon the first was reckoned, count the third, and follow it till it crosse the threed, for the quantity of it comprehended between the third term and the threed, gives the fourth proportional term required, alway remembring that every term be taken on his proper Scale.*

Here because the proportions are divers, we shall need more explication then in all the rest. Yet the variety herein, may be

be reduced to three wayes according as one of these three, either the Radius, Sine, or Tangent, doth lead in the Proportion, the three wayes are these :

1 *As the radius, is to a tangent , So is a sine to a Tangent.*
2 *As a sine, is to a tangent, So the Radius is to another tangent.*
3 *As a tangent, is to the radius; So another tangent, is to a sine.*

But this variety is not all, for each of these three wayes is subject to variation, and that upon this occasion. —— Upon the square we have no tangent greater then the radius, or tangent of 45 degrees. Wherefore the proportion must be so contrived, as that no tangent greater then of 45 gr. be ingredient into it. To that purpose serves this general direction, namely, —— If the tangent which is co-partner, in the proportion with the sine, be greater then of 45 gr. (alway provided that the two tangents doe never stand immediately together, which if they doe, may be brought into frame by transposition or alteration of the middle term.) Then, In the two first wayes the radius and sine must change places; and for the two tangents must be taken the tangents of their complements; In the third way, the co-tangents of the third and first terms must remove into the first and third places.

To shew this more particularly in the 3 former wayes.

In the first,

If the tangent required in the fourth place prove greater then of 45 gr. (which how to discover is shewed hereafter) then by the former direction this alteration must be made.

As the sine in the third place, is to the co-tangent in the second,
So is the radius in the first place, to the co-tangent of the fourth.

In the Second,

If the tangent in the second place, be greater then of 45 gr. then by the former direction this proportion must be thus changed.

As the radius in the third place, is to the co-tangent of the second.
So is the sine in the first place, to the co-tangent of the fourth.

In

In the third.

If the tangent in the third place, be greater then of 45 gr. then according to the former prescript this proportion must thus be varied.

As the co-tangent of the third place, is to the radius in the second;

So the co-tangent of the first place, is to the sine required in the fourth place.

Because in the first proportion it is unknown whether the tangent required in the fourth place be greater or lesser then of 45 gr. and yet is necessary it should be known before it can be found out, you shall therefore in practice discover it thus.

If the line whereon it is to be accounted doth not meet with the threed rightly situated upon the Square, then is it greater then of 45 gr. and then the proportion must be altered as before, but if it doe meet with the threed, then is it lesse then of 45 gr. as it should be.

Observe that the tangents are actually in the limb onely, yet may be understood to be all over the plain, for some line or other standing even against them in the plain will supply them as well as if they were actually there drawn.

And note that if such a Proportion as this do at any time happen, namely, *As a sine is to a tangent, So another sine, is to another tangent.* And that these tangents, be discovered to be one of them greater then of 45 gr. the other lesse, That then the radius is to be brought into the Proportion, by saying, *As the first sine, is to the first, So is the radius to another Tangent*; Then leaving out the first sine and tangent, and using for them the radius and this later tangent, say, As the radius is to the last found tangent, So is the sine in the third place to the tangent in the fourth; *Which Proportion suites* with those going before. But if both the tangents be either greater or lesser then of 45 gr. then may the solution be made without the help of the radius.

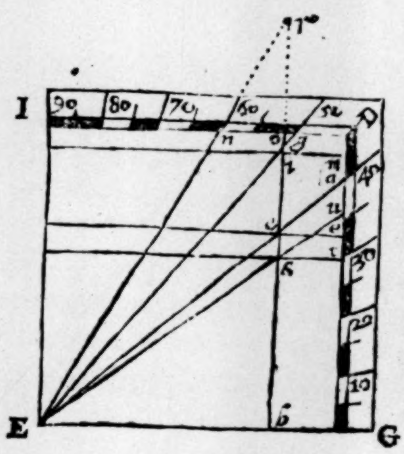
According to the former Rules generally delivered are these following Examples framed, and will fully illustrate every Case.

I. *For the first of the three several wayes* there are three cases, For either both the tangents are lesse then 45 gr. or both greater, or no lesse, the other greater.

1 As the radius is to the tangent of 40 gr. So the sine of 50 gr. to the tangent of what?

Upon the Square E I D G, I count E G for the radius, and upon the end of it in the Scales of tangents I reckon G *a*, 40 gr. thereto applying threed. Then upon E G, I count the sine of 50 gr. from E to *b*, and follow the line there arising till it cut the threed E *a*, at *c*, So that *b c* is the fourth term required, which because it is a tangent, must be numbred in the Scale of tangents, and therefore by help of the line *c e*, I transferre it thither, and find that the line lies even against 32 gr. 44 min. in the Scale of Tangents, and this is the ark of the fourth proportional term required.

2 *As the radius* is to the Tangent of 60 gr. So the sine of 50 gr. to the Tangents of what?



Upon the Square I count (as before) from G up to D and so forward to *n*, 60 gr. among the Tangents, and thereto apply the threed. Then I count the third term. E *b* the sine of 50 gr. and I follow the line there arising to the top of the Square, and yet it meets not with the threed, but beyond the Square I see that it would concurre with it at *r*, which shewes that the fourth Tangent required, is greater then the Tangent of 45 gr. and therefore standing as it doth, cannot be expressed upon the Scale of Tangents, wherefore I alter the Proportion, by transposing the first and third terms into one anothers places, and for the Tangents themselves I take their complement thus.

As the sine of 50, is to the Co-tangent of 60, So is the Radius, to a fourth Tangent, which will be the complement of that which should be produced by the former Proportion. By this alteration it comes to passe that the sine leads in the Proportion, and so this Example now fals under the Examples of the second General way, and therefore shall be resolved there.

3 *As the radius* is to the Tangent of 50 gr. So the ſine of 50 gr. to the Tangent of what?

Upon the Square I take E G for the radius, and at the end of it I reckon up to D, which is 45 gr. and ſo forward on the other ſide to *g*, that is to 50 gr. Then upon E G, I count *e b* the ſine of 50 gr. and follow the line there ariſing till it cut the threed at *i*, ſo that *b i* is the fourth term, and becauſe it is a Tangent, therefore by help of the line paſſing through *i*, that is, by the line *i m*, I transferre it to the Scale of Tangents, and find that lies even againſt 42 gr. 24 min. which is the fourth ark required.

For the ſecond of the three general wayes, there are two Caſes; For the Tangent that is Copartner with the ſine in the Proportion, may be either leſſer or greater then of 45 gr. for the leſſer, take the Example which before was preferred hither, namely, II.

1 As the ſine of 50 gr. is to the Tangent of 30 gr. So the radius is to the Tangent of what?

Firſt, upon the ſide E G, I count the ſine of 50 gr. and to the line there ariſing, I transferre G *t* the Tangent of 30 gr. by help of the line *t s*, and to *s*, I apply the threed, which threed cuts the limb in *u*, ſo that G *u* I find to be the Tangent of 37 gr. and this is the fourth term required in this Proportion; But in the ſecond Example going before, whereof this is alſo the ſolution, this 37 gr. is the complement of the fourth ark there required, ſo that the fourth ark there, ſhould be 53 gr. which becauſe it is greater then 45 gr. is therefore abſolved this way, and not the other.

2 As the ſine of 50 gr. is to the tangent of 50 gr. So the radius is to the Tangent of what?

Here becauſe the Tangent of 50 (being Co-partner in the Proportion with the ſine) is greater then the Tangent of 45 gr. and ſo cannot be expreſſed upon the ſquare, therefore the Proportion muſt be altered by changing the places of the firſt and third terms, and by taking the complement of the ſecond and fourth, after this manner.

As the radius is to the Tangent of 40 gr. So the ſine of 50 gr. to the Tangent of 32 gr. 44 min. the complement whereof 57 gr. 16 min. anſwers to the queſtion in the former Proportion, and this laſt Proportion fals under the firſt general

 way

way where the radius leads, and was resolved before in the first practice upon the Square, As E G, to G *a*, So E *b*, to *b c*, or G *c* the Tangent of 32 gr. 44 min.

III. *In the third of the three general wayes*, there are two cases, according as the Tangent of the third place, which is Co-partner in the proportion with the sine, is lesser or greater then the Tangent of 45 degrees.

1 *As the tangent* of 40 gr. is to the Radius, so the Tangent of 32 gr. 44 min. to what sine?

Upon the side G D, I count G *a*, the Tangent of 40 gr. and thereto apply the threed, then upon the same side G D, I reckon also the Tangent of 32 gr. 44 min. from G to *e*, and follow the line meeting at *e*, till it cut the threed at *c*, and the line there crossing also is *c b*, which counted from *e*, the Center of the Square, gives the sine of 50 gr. which is the fourth term required.

2 As the Tangent of 60 gr. is to the radius, So the Tangent of 53 gr. to what sine?

Here because the Tangent of 53 gr. being Co-partner in Proportion with the sine, is greater then of 45 gr. therefore the first and third terms must change places, and their complements are also to be taken, thus, As the co-tangent of 53 gr. or Tangent of 37 gr. is to the Radius, So is the co-tangent of 60 gr. or Tangent of 30 gr. to the fourth sine required.

Upon G D the side of the Square, I count G *u* the Tangent of 37 gr. thereto applying the threed. Then on the same side of the Square, I also count G *t*, the Tanegnt of 30 gr. and follow the line at *t*, till it cut the threed at *s*, and the line *s b*, there crossing being counted from *e*, the Center of the Square, gives me the Sine of 50 gr. the fourth term required.

These Examples are sufficient to give light to the rest, For no Proportion can fall out in these kinds, whereunto these Proportions and their Examples are not suitable.

And so much of Spherical Triangles.

Of

Of the use of the Square, in Right-lined Triangles.

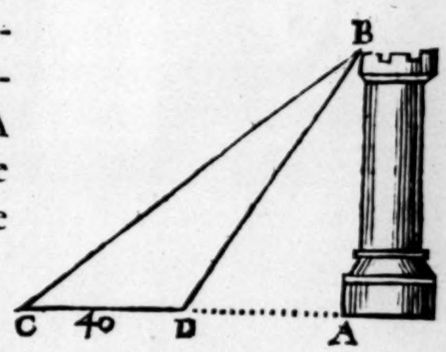
IF the Proportions be between Tangents and equal parts, then are we to use the equal parts on the sides AB, AG, *as also the larger Tangents upon the two other sides of the Square, and then the work will be the same, for form, that was before in Tangents and sines, for the lines on the superficies will carry the parts of either of these Scales to and fro, as they did before the parts of the Scales of the lesser Tangents.*

If the Proportions be between sines and equal parts, then are we to make use of the sines inscribed upon the Scales BI, CG, *together with the former equal parts, the lines upon the superficies still acting their former parts of carrying from the one to the other.*

Examples in these kinds, And first of sines, and equal parts, or Numbers.

SUppose at the two stations D C, I had observed the angles B C A, 30 gr. B D A, 50 gr. and C D the difference of Stations 40 feet, and by these observations, I require to know the altitude A B.

First, I must find the length of the lines C B, or D B, in this Example of CB, after this manner, because B C A is 30 gr. and B D A 50 grad. therefore their difference C B D is 20 gr. Now then, As the sine of C B D 20 gr.



Is to C D 40 feet,

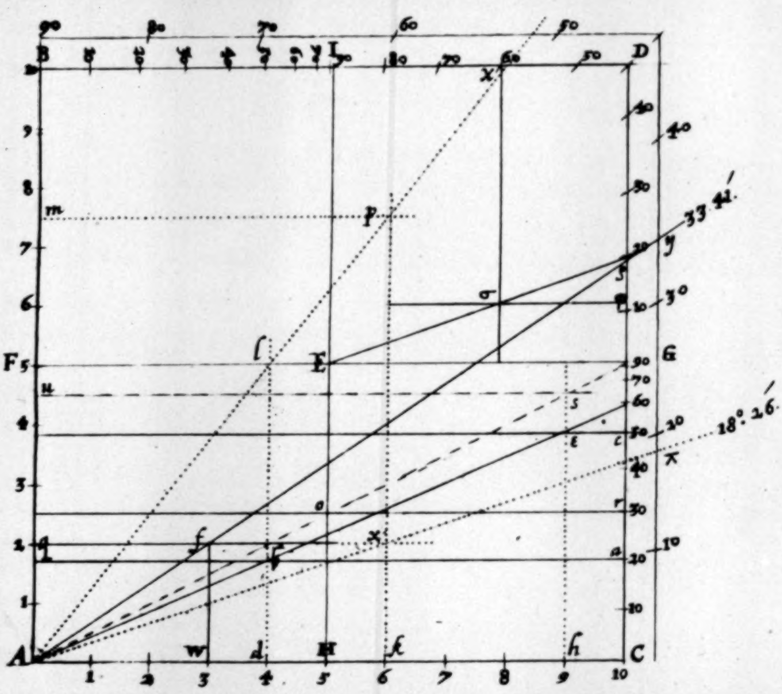
So is the sine of BDC or BDA, 50 gr.

To the length of C B required.

To resolve this upon the Square, from C, I count the line of 20 gr. to *a*, and observe the line there meeting me, then upon the side A C, I count A *d* 40 equal parts or feet, and thirdly, I reckon C *c* the third term, which is the sine of 50 gr. and follow the line there meeting me, till it crosse the threed (which was to be applied to *b*, the intersection of the lines *a b*, coming

coming from the first term, and *d b* rising from the second) at *e*, and there I find another line concurring, namely, *e b*, which I follow down to *b*, and there it shewes in the equal parts A *b* 89 feet, and 58 centesmes or hundredth parts of a foot, And this is the length of C B, now to get B A, by a second work, I say,

As C B the radius, to B A the sine of B C A 30 gr.
So B C 89 $\frac{58}{100}$ feet, to B A the altitude in feet.



To perform this Proportion, Upon the Square I take A H, equal to B I the radius, and upon H E, I count H *o*, equal to C *r* the sine of 30 gr. thereto applying the threed; Then from A to *h*, I count the length of C B, that is 89,58, and so follow the line there arising, up to the threed to *s*, where I find the line *s u*, limiting out A *u*, 44,79, that is 44 feet, and 79 centesmes of a foot, and such is the altitude of A B required.

Thus by having the three Angles of a plain Triangle, and one side you may find the two other sides; And by having two sides and an Angle opposite to one of them, you may find the other two Angles and third side, in any Right-lined Triangles whatsoever.

Examples of equal parts, and Tangents.

This kind of work may sufficiently be explained in the solution of this Probleme.

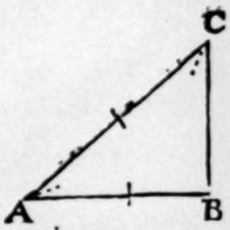
By

By having an Angle and the two sides comprehending it, to find the other Angles.

First, if the Angle comprehended be a Right-angle the work is easie.

AND here we are to use the Scales of equal parts, with the larger Tangents onely. Suppose then in the Rect-angle Triangle A B C, By having the two sides including the right angle A B 30, B C 20 parts, I would find the angles at A and C, because this Proportion holds

As A B 30, to B C 20
So A B the radius to B C,
The Tangent of B A C.



Therefore upon the Square I count A *w* 30 equal parts, and follow *w f*, till it stand even with 20 equal parts counted on the side A B, and laying the threed at *f*, I find it to cut in the limb of the greater Tangent C *y*, which is 33 gr. 41 min. And such is the quantity of the angle C A B. And the complement of it 56 gr. 19 min. is the quantity of the angle *A C B*.

Further more it is to be noted, That if by having the right angle with the two including sides, you would find the subtending side A C. In this case one of the acute angles must first be sought, and then by the Proportions of sines and equal parts, the side *A C* may be had.

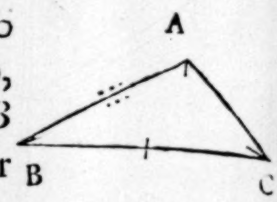
So also, If by having the distance *A B* 30 foot, and the angle *C A B* 33 gr. 41 min. I would know the *Height B C*, Upon the Square I lay the threed from C to *y* the Tangent of 33 gr. 41 min. then upon the equal parts I count *A w* 30, & follow the line rising at *w*, till it meet with the threed at *f*, and at *f*, I find the line *f q* crossing also, which followed to *q*, shewes in the limb 20 equal parts for the altitude *B C*.

By these mixt Proportions of equal parts with sines and Tangents, may all mensurations be performed, as also all conclusions upon the Common Sea-chart, with Mr. Gunters *corrections of it, to make it sufficient for* Sea-mens *use.*

Secondly,

Secondly, in any plain Triangle whatsoever.

The former Probleme may be resolved in general by this Proportion, As the sum of the two sides, is to their difference, So is the Tangent of half the sum of their opposite Angles, to the Tangent of the half difference of those Angles.



AS here, the two given parts, are *A B* 40 parts, & *BC* 20, the sum of them is 60, the difference is 20, the angle at *B* 120 gr. & therefore the sum of the two other 60 gr. the half sum of 30 gr. Now,

As 60 the sum of the sides, is to 20 their difference,
So is the Tangent of 30 gr. the half sum of the angles at A and C, to the Tangent of their half difference.

The best way for the solution of Proportions in this kind is first (as was before admonished in the joynt use of Sines and Tangents) to seek out a Tangent whereon the Radius is in Proportion as the sum of the legs is to the difference of them, which Tangent is ever lesse then the Tangent of 45 *gr. or radius, because the difference of the legs is alway lesse then the sum of them. And when the radius is brought in, the Proportion may be absolved upon the lesser Square, which is fitted for the Proportions of Sines and Tangents, in the same manner as was shewed in the like Examples before. And the Proportion will then stand thus.*

As the radius, is to this new found Tangent, So is the Tangent of half the sum of the angles, to the Tangent of half their difference.

To make it plain by the former instance, As 60 parts, are to 20; So is the radius, to what Tangent?

Upon the Scale of equal parts, I account A*k* 60 and on the line there arising I account *k x* 20, thereto applying the threed, and then I see it cut off in the greater Tangents C τ 18 gr, 26 min. which is the Tangent sought. And now that the radius is brought in, the next Proportion will be thus,

As the radius, is to the Tangent of 18 gr. 26 min.
So is the Tangent of 30 gr. half the sum of the angles, to what Tangent?

Upon

Upon the lesser Square, I take E G for the radius, and count G ϛ the tangent of 18 gr. 26 min. thereto applying the threed, then upon I D, I count I *x* the Tangent of 30 gr. and follow the line thence passing to the threed at σ, where the line σ φ shewes, in the limb the Tangent G φ, which is the Tangent of 10 gr. 54 min. the half difference of the angles required, which added to 30 gr. the half sum, makes the greater angle 40 gr. 54 min. And taken from the same 30 gr. leaveth 19 gr 6 min. for the lesser angle.

¶ *By having the three sides of a plain Triangle, to find the Angles.*

THe first work here will be to let fall a perpendicular, and to know where it will fall, and so reducing the Triangle to two Rectangles, you may resolve them as Rectangles, either by sines and equal parts, or Tangents and equal parts.

The manner of dividing a Triangle, into two Rectangles, as also to find the place where the perpendicular falls, is shewed by Mr. *Gunter* in the first Book of his *Cross-staff*, and the Proportion for the solution of it, is a proportion of equal parts or numbers onely, the manner of which is hereafter shewed in the next use of the Square in numbers or equal parts alone.

Thus farre of the use in Right-lined Triangles.

Of the use of the Square in Proportions of equal parts or Numbers only.

THe equal parts with the lines on the superficies to carry them along, will perform them very sufficiently and expeditely, If any number be too great take $\frac{1}{10}$ or $\frac{1}{100}$ part of it, and count the rest as a fraction either decimal or centesimal. And as in the former works, so here, the first & third terms, must be counted upon one side of the Instrument, the second and fourth upon the lines arising out of the terms of the former, so the threed applyed to the second, will limit out the fourth. ——

The manner of the work is alwayes a like, and may sufficiently be declared in this one Example, I would know that number whereto 60 bears the same Proportion, that 40 doth to 50: The Proportion stand thus

As 40, is to 50: So 60, to what?

Upon A C the Scale of equal parts, I count A D 40, and upon the line arising out of *d*, (by help of the Scale of equal parts upon the other side A B) I count 50 up to *l*, and thereto apply the threed, Then upon A C, I count the third term, 60 to *k*, and follow the line there arising, till it meet with the threed at *p*, and there the line *p m*, meeting also, shewes in the Scale *A B* at *m*, 75 parts, which is the fourth proportional number required. And thus in all others.

Of the use of the Square in the observation of angles.

WHen the observation is made and the sight placed, then the threed from A, applyed to the running sight, will expresse the angle in the larger Tangent, And for observing any altitude or depth, the threed alone, without the help of the running sight, will expresse the Angle, if the observation be made as usually it is by other Instruments. —— The Square at the greatest cannot observe an Angle that is greater then 90. If therefore such an Angle come to be observed, you must observe the residue of it, which is his complement to 180 degrees.

Hitherto we have had a general view of the use of the Square in all Triangles and ordinary Proportions in Numbers. Now remains the bringing of it down to particulars in every kind; which would be an infinite labour, and un-necessary to those that are any thing experienced, in the use of Instruments, especially seeing we have here a tast of every of them, and the particular Proportions are every where extant. Hereafter I may adde something more on the other side, for the present I here make stay, and content my selfe with that which hath already been deli vered.

FINIS.

OF PROJECTION.

CHAP. I.

A Description of the Horizontal Projection.

PRojections of the Sphere, are best denominated from those great Circles upon which they are projected. This here, is called Horizontal, because the Circles on it are projected upon the plain of the Horizon: the eye being placed in the Nadir point thereof, upon the superficies of the Sphere; the whole delineation of it may be deduced Geometrically out of the Horizontal Circle. But because that way is in divers respects cumbersome and not so accurate, I rather choose to make such Tables as shall better suffice for the work, which what they are, and the manner how to make them, shall now be declared.

To describe the Æquinoctial and parallels of declination, belonging thereto, two Tables as requisite especially. The first, is to tell how farre from the Center each parallel in the Projection is to cut through the Meridian, which may be called a Table of Intersections. The second, is to find how farre the Centers of those Parallels do likewise stand from the Center of the Instrument, that so they may be described; and this Table may be called, A Table of Centers. And these two Tables are variable in every Latitude.